Handout[1] 1

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1 Experiments and Events

1.1 **Definition**

An *experiment* is any process, real, or hypothetical, in which the possible outcomes can be identified ahead of time.
For an emperiment, we do not know the result in advance, so we describe the outcomes as *random*.
An *outcome* is a specific result. For example, an outcome for a coin flip can be either Head or Tail. Rolling a six-sided die has six outcomes {1,2,3,4,5,6}.
The *sample space* is the set of all possible outcomes. For example, if two coins are flipped, the sample sapace is $\mathcal{S}=$ {HH,HT,TH,TT}
An *event* is a well-defined set of possible outcomes of the experiment. It is a subset of outcomes in sample space.

1.2 **Example**

Figure 1: Magic Board

There is a magic board on the 4th floor at PCL. Couples draw a love heart and write their names inside it to prey for permanent romance. If we suppose there exist only two outcomes: married and not married, then drawing a heart can be considered as an *experiment* for the magic board. And an *event* can be a set of combinations of outcomes, for example, all of them get married.

Suppose based on the existed 50 observations, 45 of them got married after drawing the heart, how will you describe the magic level of the magic board? How confident are you to draw your heart and write down your and his/her names?

Suppose based on the existed 50 observations, 2 of them got married and 48 of them broke up, how will you describe the magic level? How willing are you to prey to the magic board?

# of observations	# of married	magic level	your decision
50	45	90%	
50	2	4%	

The essential purpose of statistics is to figure out some parameters which can help us to make decisions.

[1]This handout is made by Hongkai Wang for Econ 329 Economic Statistics. It is adapted from Professor Wiseman's lectures, however, all errors are mine.

2 Set Theory

2.1 **Definition**

For events A and B,

1. A is a *subset* of B, or A is *contained in* B, written as $A \subset B$, if every element in A is an element in B.
2. The event with no outcomes, denoted as $\emptyset$, is called the *null* or *empty set*.
3. A is *finite* if it contains finitely many elements.
4. An infinite set A is *countable* if there is a one-to-one correspondence between the elements of A and the set of natural numbers {1,2,3,...}.
5. The *union* $A \cup B$ is the collection of all outcomes that are either in A or in B, or both.
6. The *intersection* $A \cap B$ is the collections of outcomes that are in both A and B.
7. The *complement* A^{C} of A are the outcomes in $\mathcal{S}$ that are not in A.
8. The event A and B are *disjoint* if they have no outcomes in commom: $A \cap B = \emptyset$.

2.2 **More Definitions**

1. The *union* of n sets $A_1, A_2, A_3, \ldots, A_n$ is defined to be the set of outcomes that belong to at least one of these sets, denoted as $\bigcup_{i=1}^{n} A_i$.
2. The *intersection* of n sets $A_1, A_2, A_3, \ldots, A_n$ is defined to be the set that contains the elements that are commom to all these sets, denoted as $\bigcap_{i=1}^{n} A_i$.
3. The sets $A_1, A_2, A_3, \ldots, A_n$ are *disjoint* if for every $i \neq j$, we have A_i and A_j are disjoint.

2.3 **Theorem**

1. Let A, B, and C be events,
   a). If $A \subset B$ and $B \subset A$, then $A = B$.
   b). If $A \subset B$ and $B \subset C$, then $A \subset C$.
2. For any event A, $\emptyset \subset A$.
3. Let A be any event in the sample space $\mathcal{S}$, then: $(A^{\mathsf{C}})^{\mathsf{C}} = A$, $\emptyset^{\mathsf{C}} = \mathcal{S}$, and $\mathcal{S}^{\mathsf{C}} = \emptyset$.
4. For sets A and B,
   $A \cup B = B \cup A$, $A \cup A = A$, $A \cup A^{\mathsf{C}} = \mathcal{S}$, $A \cup \emptyset = A$, $A \cup \mathcal{S} = \mathcal{S}$.
5. For sets A and B,
   $A \cap B = B \cap A$, $A \cap A = A$, $A \cap A^{\mathsf{C}} = \emptyset$, $A \cap \emptyset = \emptyset$, $A \cap \mathcal{S} = A$.
6. If $A \subset B$, then $A \cup B = B$, and $A \cap B = A$.
7. *Associative*:
   For sets A, B, and C, the associative property says $A \cup B \cup C = (A \cup B) \cup C = A \cup (B \cup C)$.
   Similarly, $A \cap B \cap C = (A \cap B) \cap C = A \cap (B \cap C)$.
8. *Distributive*:
   For sets A, B, and C, the distributive property says $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, and
   $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
9. *Partition Theorem*:
   For two sets A and B, $A \cap B$ and $A \cap B^{\mathsf{C}}$ are disjoint, and $A = (A \cap B) \cup (A \cap B^{\mathsf{C}})$.
   In addition, B and $A \cap B^{\mathsf{C}}$ are disjoint and $A \cup B = B \cup (A \cap B^{\mathsf{C}})$.
10. *De Morgan's Law*:
    For sets A and B, $(A \cup B)^{\mathsf{C}} = A^{\mathsf{C}} \cap B^{\mathsf{C}}$, and $(A \cap B)^{\mathsf{C}} = A^{\mathsf{C}} \cup B^{\mathsf{C}}$.

Try to prove the theorems by yourself! Hint: Use Venn Diagrams for some of them.